

PDE Dynamics: An Introduction. By *Christian Kuehn*. SIAM, Philadelphia, 2019. \$69.00. xiv+267 pp., softcover. *Mathematical Modeling and Computation*. Vol. 23. ISBN 978-1-611975-65-9.

The stated objective of the book is to promote the interchange of ideas between the study of dynamical systems and partial differential equations (PDEs). The author notes that in recent years, the two fields have drifted apart, and this book is an attempt to (re)promote the possibilities of dynamical systems ideas in the study of PDEs as well as the impact that PDE methods can have on the study of dynamical systems.

There is an extended preface, focused on course design, which reveals another purpose of the book—to serve as a road map for a researcher reasonably well versed in PDEs from a dynamical systems point of view to use in a one- or two-semester graduate course/seminar series/summer school on the topics. There are quite a few permutations of potential courses/seminars listed as options, many of which seem like interesting courses to either give or sit in on. The nature of the book, however, makes it reasonably apparent that this text can only serve as a road map or starting point, but the book does provide many, many additional references and readings. Indeed, this is one of my favorite aspects, namely, that

there is so much further reading and additional references. Moreover, it seems that much care has been given as to where to go next on any given topic, and the end of each chapter has a nice succinct summary of the literature (or at least the author's path through it).

The book has a nice blend of the abstract along with concrete examples, which I think greatly helps to get its points across. This also mitigates the fact that the book is, by design, somewhat light on depth. Many topics are covered—all at quite a high level, which makes for enjoyable reading, and the book has 36 short, digestible chapters and two appendices. Chapters begin with a motivating discussion before moving on to the benchmark equation for the chapter—the paradigm equation illustrating the dynamics the author wants to emphasize. Most chapters are focused on studying specific aspects of PDE dynamics through these benchmark equations, and this gives a nice, tangible anchor on which to focus one's thoughts, while still making it clear where abstraction is possible.

Chapter 1 opens with a rapid introduction, with subsequent chapters providing a quick review of the background material in geometric dynamical systems and functional analysis, before the book moves on to bifurcations of solutions, as well as stability and spectral theory. The topic then shifts to existence of traveling wave solutions and some of their dynamics before returning again to spectral stability and the Evans function. Chapters 12 and 13, somewhat of a diversion, introduce the topics of multiple scales and a discussion on the validity of amplitude equations. Chapter 14 returns to PDE dynamics with an introduction to semigroup theory, with the subsequent chapters addressing further themes of geometric dynamics in PDEs such as dissipation and absorbing sets, invariant manifolds, and attractors. The rest of the book is dedicated to multiscale/asymptotic topics, variational methods, and dynamics in Hamiltonian systems. The appendices briefly cover the numerical methods of finite differences, finite elements, and continuation.

I enjoyed the author's philosophical asides as well as descriptions of somewhat

apocryphal knowledge (what the author calls “folklore”). Some were new to me, while others were facts with which I was already familiar. The passing on of the philosophy of this subject, as well as the folklore, is quite refreshing, as it is something that I find is often lost in textbooks, where the emphasis is so much on rigor that oftentimes the reasoning behind the stating of a theorem gets a bit lost.

This would be a nice book to use as the starting point for a course. There is a wealth of topics to choose from, they are introduced intuitively and presented in the context of nice pedagogical examples, and, as already mentioned, there are plenty of suggestions and helpful tips about where to find additional resources.

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